MATH 16A MIDTERM 1(002)

PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and section: ______________________________

GSI's name: ______________________________

Blank

Blank

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) Determine the domains of the following functions:

(a)

$$\frac{1}{\ln(x)}$$

Solution:

$\ln(x) = 0 \iff x = 1$

$\Rightarrow$ Domain of $\frac{1}{\ln(x)}$ is $x > 0$ and $x \neq 1$

i.e. $(0,1) \cup (1,\infty)$

(b)

$$\sqrt{\frac{3x-1}{1-x}}$$

Solution:

$3x - 1 \geq 0$, $1 - x > 0$ $\Rightarrow$ $x \geq \frac{1}{3}$, $1 > x$ $\Rightarrow$ $\frac{1}{3} \leq x < 1$

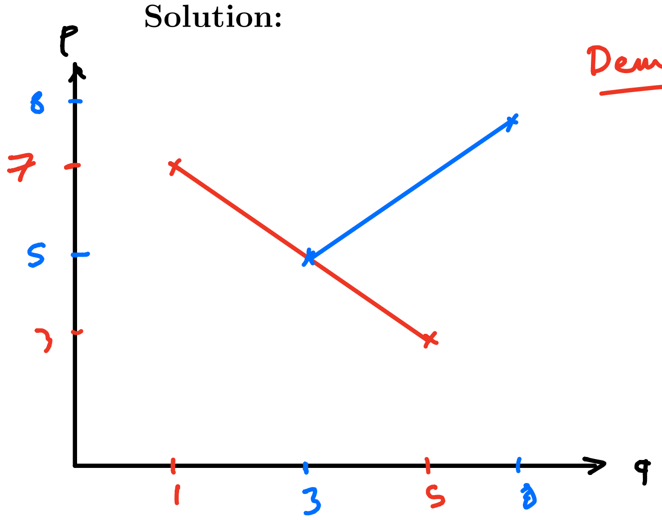
$3x - 1 \leq 0$, $1 - x < 0$ $\Rightarrow$ $x \leq \frac{1}{3}$, $1 < x$ $\Rightarrow$ No possible x

$\Rightarrow$ Domain is $[\frac{1}{3}, 1)$.

2. (25 points) A product is to be supplied and sold. If the price per unit is 7 dollars the demand 1 unit. If the price per unit is 3 dollars the demand 5 units. If the price per unit is 8 dollars the supplier is willing to provide 8 units. If the price per unit is less than 5 dollars there will be a shortage. If it is more than 5 dollars there will be a surplus.

   (a) Determine the supply and demand equations. Hint: determine the demand equation first.

   Solution:



Demand : Slope $= \frac{3-7}{5-1} = -1$

$p - 7 = -1(q-1)$

$\Rightarrow \quad p = 8 - q = D(q)$

equilibrium

$5 = 8 - q \Rightarrow q = 3$

Supply : Slope $= \frac{8-5}{5} = \frac{3}{5}$

$p - 5 = \frac{3}{5}(q-3)$

$\Rightarrow \quad p = \frac{3}{5}q + \frac{16}{5} = S(q)$

   (b) What is the minimum price per unit needed to guarantee that a supplier will provide one unit?

$S(1) = \frac{3}{5} + \frac{16}{5} = \frac{19}{5}$ dollars.

3. Calculate the following limits. If they do not exist determine if they are ∞ or $-\infty$.

(a)

$$\lim_{x\to 3} \ln(\frac{x^2+1}{x^2+3})$$

Solution:

$$\lim_{x\to 3} \ln\left(\frac{x^2+1}{x^2+3}\right) = \ln\left(\frac{3^2+1}{3^2+3}\right)$$

(b)

$$\lim_{x\to\infty} e^{\frac{1-x^5}{2+x+3x^5}}$$

Solution:

Both degree 5

$$\lim_{x\to\infty} e^{\left(\frac{1-x^5}{2+x+3x^5}\right)} = e^{\left(\lim_{x\to\infty} \frac{1-x^5}{2+x+3x^5}\right)} = e^{-\frac{1}{3}}$$

(c)

$$\lim_{x\to -2^+} \frac{x^2+3x+2}{x^2+4x+4}$$

Solution:

$$\lim_{x\to -2^+} \frac{x^2+3x+2}{x^2+4x+4} = \lim_{x\to -2^+} \frac{(x+2)(x+1)}{(x+2)(x+2)} = \lim_{x\to -2^+} \frac{x+1}{x+2}$$

$$\lim_{x\to -2^+} x+1 = -1 < 0$$

$$\Rightarrow \lim_{x\to -2^+} \frac{x+1}{x+2} = -\infty \quad \text{(DNE)}$$

$x+2<0$ | -2 | $x+2>0$

$$\Rightarrow \lim_{x\to -2^+} x+2 = 0^+$$

4. Let $f(x) = \begin{cases} \frac{x^2+kx-2}{x^2+2x-3} & \text{if } x \neq 1 \\ l & \text{if } x = 1 \end{cases}$ for some real number k and l.

Determine what values of k and l make $f(x)$ continuous at $x = 1$? Carefully justify why $f(x)$ is continuous at $x = 1$ for these values.

Solution:

$$\lim_{x \to 1} x^2 + kx - 2 = 1^2 + k - 2 = k - 1$$

$$\lim_{x \to 1} x^2 + 2x - 3 = 1^2 + 2 \cdot 1 - 3 = 0$$

$$k - 1 \neq 0 \Rightarrow \lim_{x \to 1} \frac{x + kx - 2}{x^2 + 2x - 3} \quad \text{DNE}$$

$$k - 1 = 0 \Rightarrow k = 1$$

$$\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 + 2x - 3} = \lim_{x \to 1} \frac{(x+2)(x-1)}{(x+3)(x-1)}$$

$$= \lim_{x \to 1} \frac{x+2}{x+3} = \frac{3}{4}$$

$\Rightarrow$ f continuous at $x = 1$ if $k = 1$

and $l = \frac{3}{4}$.

You may assume $x \neq 1$

5. Using limits, calculate the derivative of $f(x) = \frac{x-1}{\sqrt{x}-1}$. Is there anywhere in the graph $y = f(x)$ with tangent line perpendicular to $y = 2x + 3$?

Solution:

$$f(x) = \frac{x-1}{\sqrt{x}-1} = \frac{(\sqrt{x}+1)(\sqrt{x}-1)}{\sqrt{x}-1} = \sqrt{x}+1$$

$$\Rightarrow \quad f'(x) = \lim_{h \to 0} \frac{(\sqrt{x+h}+1) - (\sqrt{x}+1)}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x}+\sqrt{x+h})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x}+\sqrt{x+h}}$$

perpendicular slope

$$= \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{-1}{2} \quad \Rightarrow \quad \frac{-1}{2} = \frac{1}{2\sqrt{x}} \Rightarrow \sqrt{x} = -1$$

(No solutions)

$\Rightarrow$ There is nowhere on graph $y = f(x)$ with tangent line perpendicular to $y = 2x+3$.

END OF EXAM

Blank

Blank

Blank